Further Pure 1

Complex Numbers
Exercise B

9(i) $(z-\alpha)(z-\beta) = z^2 - \alpha z - \beta z + \alpha\beta$
$= z^2 - (\alpha+\beta)z + \alpha\beta$

The quadratic equation with roots α and β can be written as $(z-\alpha)(z-\beta) = 0$

$\Rightarrow z^2 - (\alpha+\beta)z + \alpha\beta = 0$

(ii) (a) Sum of roots $= 7+4j+7-4j = 14$

Product of roots $= (7+4j)(7-4j) = 49+16 = 65$

Equation is $z^2 - 14z + 65 = 0$

(b) Sum of roots $= \frac{5j}{3} - \frac{5j}{3} = 0$

Product of roots $= \frac{5j}{3} \times -\frac{5j}{3} = \frac{25}{9}$

Equation is $z^2 + 0z + \frac{25}{9} = 0$

i.e. $9z^2 + 25 = 0$

(c) Sum of roots $= -2+\sqrt{8}j - 2 - \sqrt{8}j = -4$

Product of roots $= (-2+\sqrt{8}j)(-2-\sqrt{8}j) = 4+8 = 12$

Equation is $z^2 + 4z + 12 = 0$

(d) Sum of roots $= 2+j+3+2j = 5+3j$

Product of roots $= (2+j)(3+2j)$
$= 6+4j+3j-2 = 4+7j$

Equation is $z^2 - (5+3j)z + 4+7j = 0$