

# Further Pure 1

## Complex Numbers

### Exercise C

$$4. (i) z = 3 + 4j$$

$$|z| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$(ii) w = 5 - 12j$$

$$|w| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

$$(iii) zw = (3 + 4j)(5 - 12j) = 15 - 36j + 20j + 48 \\ = 63 - 16j$$

$$|zw| = \sqrt{63^2 + 16^2} = \sqrt{4225} = 65$$

$$(iv) \frac{\bar{z}}{w} = \frac{3 + 4j}{5 - 12j} = \frac{(3 + 4j)(5 + 12j)}{(5 - 12j)(5 + 12j)} \\ = \frac{15 + 36j + 20j + 48}{25 + 144} \\ = \frac{-33 + 56j}{169}$$

$$\left| \frac{\bar{z}}{w} \right| = \frac{1}{169} \sqrt{33^2 + 56^2} = \frac{\sqrt{4225}}{169} = \frac{65}{169} = \frac{5}{13}$$

$$(v) \frac{w}{z} = \frac{5 - 12j}{3 + 4j} = \frac{(5 - 12j)(3 - 4j)}{(3 + 4j)(3 - 4j)} \\ = \frac{15 - 20j - 36j - 48}{9 + 16} \\ = \frac{-33 - 56j}{25}$$

$$\left| \frac{w}{z} \right| = \frac{1}{25} \sqrt{33^2 + 56^2} = \frac{\sqrt{4225}}{25} = \frac{65}{25} = \frac{13}{5}$$

$$|zw| = |z||w|, \quad \left| \frac{\bar{z}}{w} \right| = \frac{|z|}{|w|}, \quad \left| \frac{w}{z} \right| = \frac{|w|}{|z|}$$